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assuming that it is unaffected by the arms, is 1,176 lbs. per sq. in. If the safe stress permissible in the material is 6,000 lbs. per sq. in., show that the greatest speed at which the wheel can be run with safety is about 225 revolutions per minute.

Solution by William W. Johnson, Cleveland, Ohio.

The unit stress in the flywheel rim due to centrifugal force is found as follows: In the general equation of force, $F = Ma$, the acceleration a in the present instance is v^2/r, in which v = velocity of rim in feet per second, R = radius of rim in feet, W = weight of rim in pounds, and g = acceleration due to gravity. Hence, centrifugal force $= F = Mv^2/R = Wv^2/gR$.

The resultant of half of this force tends to disrupt one half of the rim from the other half. This rupture is resisted by the two sections of the rim at each end of the diameter.

The total tension T in a cross-section of the rim is to one half the sum of half of the radial forces as the diameter of the flywheel is to half its circumference. Therefore,

$$T = F/2\pi = Wv^2/2\pi Rg.$$

Let $W = 2\pi RAw/144$, in which A = area of cross-section of rim in square inches, and w = weight of rim material in pounds per cubic foot.

Then, $T = Awv^2/144g$. Let $T = AS$, where S = stress per unit area of cross-section of rim. Then, $S = wv^2/144g$.[1]

Put $v = \pi Dn/60$, where D = mean diameter of rim in feet, and n = number of revolutions per minute, we have

$$S = \frac{w(\pi Dn)^2}{60^2 \times 144g}. \tag{A}$$

Solving for n, we have

$$n = \frac{720}{\pi D}\sqrt{\frac{Sg}{w}}. \tag{B}$$

Putting, $D = 21$, $n = 100$, $w = 448$, $S = 6{,}000$, and $g = 32.16$ in formulas (A) and (B) we find

Answer (a): $S = 1{,}170$ lbs. per sq. in., and

Answer (b): $n = 226$ revolutions per min.

Also solved by J. B. Reynolds.

334. Proposed by HORACE OLSON, Chicago, Illinois.

A particle of elasticity e is projected with velocity v at an angle φ with a plane inclined to the horizontal at an angle ψ; its plane of motion is perpendicular to the inclined plane. Show that after $2v \sin \varphi/g(1 - e) \cos \psi$ seconds it will cease to rebound and will move along the plane with an initial velocity $v \cos \varphi - 2v \sin \varphi \tan \psi/1 - e$ and a uniform acceleration $g \sin \psi$ down the plane.

Solution by Jos. B. Reynolds, Lehigh University.

The acceleration of the particle may be resolved into a component of $g \cos \psi$ perpendicular to the plane and a component $g \sin \psi$ down the plane. Let v_1 be the initial component of the velocity of the particle perpendicular to the plane, v_2 the component of velocity perpendicular to the plane after the first rebound, v_3 after the second rebound, etc. Also, let t_1 be the time from the instant of projection until the first impact, t_2 the time from the first impact until the second, t_3, from the second until the third, etc. Then

$$v_2 = ev_1, \qquad v_3 = ev_2 = e^2v_1, \quad \text{etc.}$$

$$t_1 = \frac{2v_1}{g \cos \psi}, \qquad t_2 = \frac{2v_2}{g \cos \psi}, \qquad t_3 = \frac{2v_3}{g \cos \psi}, \quad \text{etc.}$$

So that if t is the time before the ball ceases to rebound,

$$t = t_1 + t_2 + t_3 + \cdots = \frac{2v_1}{g \cos \psi}(1 + e + e^2 + \cdots + e^n)_{n=\infty} = \frac{2v_1}{(1 - e)g \cos \psi}$$

$$= \frac{2v \sin \varphi}{(1 - e)g \cos \varphi}, \qquad \text{since} \qquad v_1 = v \sin \varphi.$$

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[1] S is independent of the radius and depends only on the rim velocity.

Since the velocity along the plane is not affected by the impacts the velocity v' at the end of time, t, will be upward along the plane end of magnitude

$$v' = v \cos \varphi - g \sin \psi t$$

$$= v \cos \varphi - \frac{2v \sin \varphi \tan \psi}{(1 - e)}, \quad \text{since} \quad t = \frac{2v \sin \varphi}{(1 - e) g \cos \psi}.$$

Also solved by the PROPOSER.

335. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

A heavy particle is projected upwards with a velocity V in a medium resisting as the nth power of the velocity. Prove that the elevation of the particle when the velocity downwards is V is equal to LT, where L is the limiting velocity and T is the time in which the particle falling from rest in the medium will acquire a velocity V^2/L.

SOLUTION BY JOS. B. REYNOLDS, Lehigh University.

As the solution will show, this problem should read as follows: A heavy particle is projected upwards with a velocity V in a medium resisting as the nth power of the velocity. Prove that the whole space (up and down) described when the downward velocity is V is equal to LT, where L is the limiting velocity and T is the time in which the particle falling from rest in the medium will acquire a velocity V^2/L.

The equation of motion for the particle upward is

$$(1) \qquad -v\frac{dv}{ds} = g + \mu v^n$$

μ being the proportionality factor, and the equation of motion for the particle downward is

$$(2) \qquad v\frac{dv}{ds} = g - \mu v^n = \frac{dv}{dt}.$$

When the particle has acquired the limiting velocity L its acceleration is zero. So by equation (2) we have $g/\mu = L^n$. Also by (2), when falling in the medium,

$$dt = \frac{dv}{g - \mu v^n}, \quad \text{and} \quad \therefore \ T = \int_0^{V^2/L} \frac{dv}{g - \mu v^n} = \frac{1}{\mu}\int_0^{V^2/L} \frac{dv}{L^n - v^n};$$

whence,

$$(3) \qquad LT = \frac{L^{n+1}}{g}\int_0^{V^2/L} \frac{dv}{L^n - v^n}.$$

From (1), in the upward motion,

$$ds = -\frac{vdv}{g + \mu v^n};$$

whence, if the particle rises to a height h before coming to rest,

$$h = -\int_V^0 \frac{vdv}{g + \mu v^n} = \int_0^V \frac{vdv}{g + \mu v^n}.$$

When falling, by (2),

$$ds = \int \frac{vdv}{g - \mu v^n},$$

so that if it has fallen a distance h_1 when the velocity is V, we have

$$h_1 = \int_0^V \frac{vdv}{g - \mu v^n}.$$

Then the total distance described (up and down) is

$$h + h_1 = \int_0^V \left(\frac{1}{g - \mu v^n} + \frac{1}{g + \mu v^n}\right) vdv = \int_0^V \frac{2gvdv}{g^2 - \mu^2 v^{2n}},$$

or

$$h + h_1 = \frac{g}{\mu^2}\int_0^V \frac{2vdv}{L^{2n} - V^{2n}} = \frac{g}{\mu^2 L^n}\int_0^V \frac{2vdv}{L^n - (V^2/L)^n}.$$